Exploiting Constructive Interference for Scalable Flooding in Wireless Networks

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***Abstract*—Exploiting constructive interference in wireless networks is an emerging trend for it allows multiple senders transmit an identical packet simultaneously. Constructive interference based flooding can realize millisecond network flooding latency and sub-microsecond time synchronization accuracy, require no network state information and adapt to topology changes. However, constructive interference has a precondition to function, namely, the maximum temporal displacement Δ of concurrent packet transmissions should be less than a given hardware constrained threshold. We disclose that constructive interference based flooding suffers the scalability problem. The packet reception performances of intermediate nodes degrade significantly as the density or the size of the network increases. We theoretically show that constructive interference based flooding has a packet reception ratio (PRR) lower bound (95.4%) in the grid topology. For a general topology, we propose the spine constructive interference based flooding (SCIF) protocol. With little overhead, SCIF floods the entire network much more reliably than Glossy [1] in high density or large-scale networks. Extensive simulations illustrate that the PRR of SCIF keeps stable above 96% as the network size grows from 400 to 4000 while the PRR of Glossy is only 26% when the size of the network is 4000. We also propose to use waveform analysis to explain the root cause of constructive interference, which is mainly examined in simulations and experiments. We further derive the closed-form PRR formula and define interference gain factor (IGF) to quantitatively measure constructive interference.**

I. Introduction

Network flooding is a fundamental service in wireless ad-hoc networks for many purposes, such as data dissemination [2], time synchronization [3], the creation of data collection tree [4], and various applications [5] [6]. The main objective of network flooding is to propagate packets reliably and as fast as possible. By leveraging link characteristics such as link correlation [7], link dynamics [8] and link quality [9], current approaches focus on identifying which nodes to relay packets. Those approaches suffer large overhead to maintain the network state. By exploring properties of wireless radios such as the capture effect [10] and implementing controlled concurrency, Flash [11] achieves rapid network flooding with 2s latency for 90% reliability in a network of 48 telosB motes. Glossy [1] realizes by far the fastest packet propagations across an entire network. By employing a concurrent transmission technique called constructive interference [12], Glossy achieves a magnitude of millisecond flooding latency and sub-microsecond time synchronization accuracy per hop.

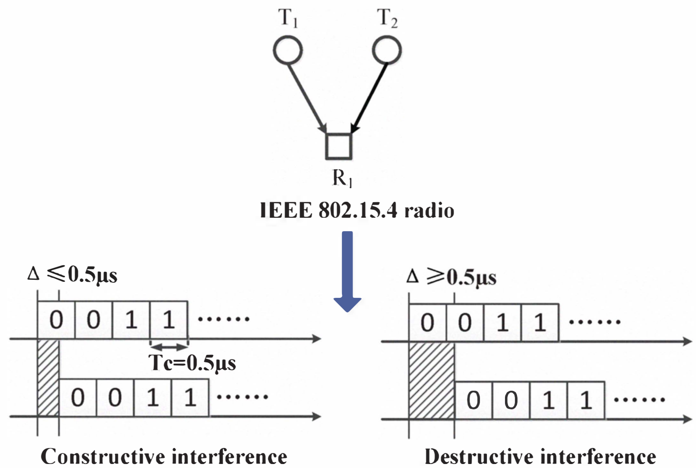

Fig. 1. Concurrent transmissions of an identical packet in IEEE 802.15.4 radio.

Recently employed in Backcast [12], constructive interference can alleviate the ACK storm problem [13], reduce the transmission latency of acknowledge packets, and improve the reliability of packet transmissions. Constructive interference originates from the physical layer tolerance for multi-path signals: when multiple senders transmit an identical packet simultaneously, rather than cause mutual interference, concurrent packet transmissions can improve the packet reception rate (PRR) of a common receiver. However, constructive interference has a precondition that requires the maximum temporal displacement Δ of concurrent packet transmissions should be less than a threshold duration. Here, the threshold is constrained by physical layer designs and equals to $0.5\mu s$ for IEEE 802.15.4 compatible receivers. Fig. 1 illustrates the physical layer phenomena for IEEE 802.15.4 radio.

The phenomena of constructive interference is previously examined in simulations [1] and experiments [14]. In this paper, we explore the root cause of constructive interference with waveform analysis. We then disclose the scalability problem for constructive interference based flooding, since PRR decreases

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significantly when the density or the size of the network grows. With theoretical analysis, we show that a network with grid topology can efficiently resist packet collisions induced by scalable flooding with constructive interference. For a general topology, we further propose the spine constructive interference based flooding (SCIF) protocol, which first constructs the spine of a given topology, and then conducts network flooding on the spine in the same way as Glossy [1]. The key difference is that, the ordinate nodes connecting to the spine only receive the flooding packets, without retransmitting those packets. Extensive simulations show that the PRR of SCIF is much higher than that of Glossy in high density and large-scale wireless networks. It is deserved to mention that, we take IEEE 802.15.4 radio as an example in this discussion, and the analysis can be easily extended to other radios.

The major contributions of this paper can be summarized as follows.

i) We show the root cause of constructive interference with waveform analysis. Moreover, we derive the closed-form PRR formula, and define interference gain factor (IGF) to quantitatively characterize constructive interference.

ii) To the best of our knowledge, we are the first to disclose the scalability problem of constructive interference based flooding. We show that constructive interference based flooding is reliable in the grid topology, and has a lower bound (95.4%) of PRR, validated by simulations.

iii) We propose the SCIF protocol which is more reliable than Glossy by constructing a virtual backbone of a given topology. Simulations show that the PRR of SCIF keeps stable above 96% as the network size grows from 400 to 4000 while the PRR of Glossy is only 26% when the network size is 4000.

The rest of this paper is organized as follows. Section II presents the related work, followed by waveform analysis of constructive interference for IEEE 802.15.4 radio in section III. Section IV introduces the mechanism for constructive interference based flooding, and reveals the scalability problem. The design of the SCIF protocol is described in section V. Section VI provides the simulation results. We conclude the work in Section VII.

II. Related Work

Exploiting concurrent transmissions over interference in wireless networks is a promising trend, for its ability to decode packets from collisions, so as to increase network throughput [15], to alleviate the broadcast storm problem of ackowledgements [13], to enhance packet transmission reliability, and to reduce flooding latency [11]. Prior works mainly focus on exploring wireless radio properties such as the capture effect [10] and message-in-message (MIM) [16], both of which do not require nodes to concurrently transmit the same packet. However, techniques leveraging the capture effect or the MIM physical phenomena either require the strong signal arriving first or need special hardware support to continuously search for the stronger signal. Differing from the capture effect and MIM, constructive interference stems from the physical layer tolerance for multi-path signals. Constructive interference is experimentally discovered by Dutta et al. [14], who explore concurrent transmissions of short acknowledgment packets automatically generated by the radio hardware, to alleviate the ACK implosion problem [13].

The proposed SCIF protocol is related to prior work on network flooding, which is a fundamental service in wireless networks. Previous works like CF [7] and RBP [9] improve network flooding performance by leveraging link characteristics and identifying which nodes to relay packets. Those protocols require nodes to maintain the working states of nearby neighbors, introducing huge overhead. Opportunistic flooding [8] can efficiently reduce flooding latency and redundancy by using links outside the energy optimal tree to forward opportunistically early packets. Those methods [7]–[9] all rely on CSMA/CA protocol for MAC layer access and collision avoidance, while SCIF exploits constructive interference, a concurrent transmission mechanism. By exploring the capture effect and utilizing controlled concurrency techniques, Flash [11] can realize rapid network flooding with 2s latency for 90% reliability. Flash requires the stringent power control to guarantee PRR and its flooding performance degrades significantly as the density or the size of the network increases. SCIF can also benefit from the capture effect. By implementing elaborate designs such as the compensation of MCU irregular instructions and the disablement of irrelevant interrupts as well as hardware timers, Glossy [1] realizes precise timing to control multiple senders to transmit packets simultaneously. Therefore, Glossy employs constructive interference to achieve a magnitude of millisecond flooding latency of data (not acknowledgement). Nevertheless, as disclosed in this paper, Glossy suffers the scalability problem, namely, the PRR performance of Glossy degrades significantly as the density or the size of the network increases. The main objective of SCIF is to address this problem.

We disclose the root cause of constructive interference with waveform analysis, which was previously observed in simulations and experiments. This work is related to the theoretical analysis of the delayed replica of a modulated signal [17]. The contribution of this paper is, we derive the closed-form PRR formula for packet transmissions with constructive interference. We also define IGF to quantitatively measure the improved performance due to constructive interference. Theoretical analysis is validated by extensive simulations.

Our design SCIF is also related to prior work on topology control. There are a number of examples, which utilize topology control to realize efficient routing in wireless networks [18]. The most common techniques of topology control are locally constructing connected dominating set (CDS). SCIF proposes a lightweight approach to construct the spine of a network. Being approximately considered as a grid topology, the spine structure is shown to be able to efficiently resist packet collisions due to scalable flooding with constructive interference. SCIF adopts the proposed lightweight topology control method to solve the scalability problem.

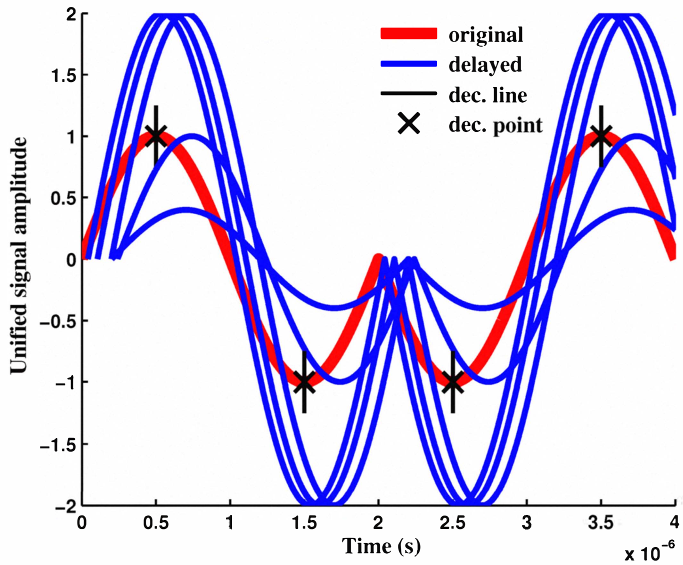

Fig. 2. Constructive interference (maximum temporal displacement $\Delta \leq T_c$).

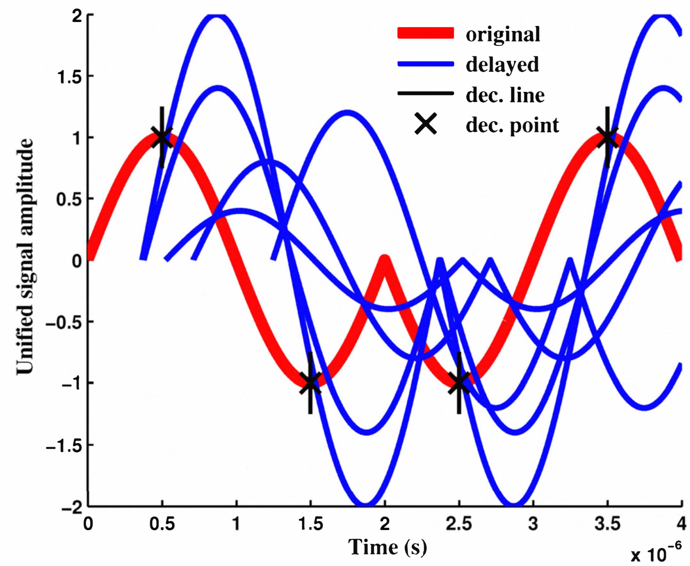

Fig. 3. Destructive interference (maximum temporal displacement $\Delta > T_c$).

III. Theoretical Waveform Analysis

A. Concurrent Transmissions

Concurrent transmissions have been studied extensively as they can help receivers to decode packet contents from collisions. Constructive interference originates from the scenario that multiple transmitters send an identical packet to a common receiver simultaneously. Interference is constructive if it *helps* the common receiver to decode the original signal. By contrast, interference is destructive if it *prevents* the common receiver from accurately decoding the superimposed signals. From Fig. 1, it can be observed that constructive interference requires the same waveform being transmitted within a threshold period T_c. Indeed, the period T_c characterizes the physical layer tolerance for multi-path signals. If the maximum temporal displacement Δ exceeds the threshold period, the common receiver *might* not be able to decode the packet, which results in collisions. Strictly speaking, receivers might also decode packet transmissions under destructive interference. In the following PRR modeling and theoretical analysis, successful packet receptions under destructive interference are omitted for simplicity, if not particularly indicated.

B. Waveform Analysis

We first illustrate the root cause of constructive interference with waveform analysis. In Fig. 2, a 4-chips ([1 0 0 1]) MSK signal with 5 replicas is received by a common IEEE 802.15.4 compatible receiver. For simplicity, the original signal is assumed to has unit amplitude and zero phase offset. Amplitudes and phase offsets of the 5 replicas are uniformly distributed in $[0,2]$ and $[0,0.5]\mu$s respectively. It can be observed from Fig. 2 that the original signal has the same signs as the 5 replicas at critical time points $(2n+1)T_c$ $(n = 0,1,2,...)$. Rather than resulting in mutual interference, the 5 replicas *help* the receiver decode the original signal. While in Fig. 3, when the maximum temporal displacement Δ among those transmitted signals exceeds one chip period $T_c(0.5\mu s)$, the 5 replicas might have opposite signs with the expected signal at those critical decision time points, leading to signal overlapping.

With waveform analysis illustrated in Fig. 2 and Fig. 3, in the following part of this subsection, we analyze the packet reception performance of IEEE 802.15.4 compatible receivers under constructive interference, derive the closed-form PRR formula, and define IGF to measure constructive interference.

The basic principle of 802.15.4 PHY layer is elaborated in [19]. Let $S_{msk}(t)$ be the transmitted signal after MSK modulation, $I(t)$ and $Q(t)$ denote the in-phase component and quadrature-phase component respectively. Let $\omega_c = \pi/2T_c$ represent the radial frequency of half-sine pulse shaping. The combined MSK signal can mathematically be calculated as

$$S_{msk}(t) = I(t)\sin\omega_c t - Q(t)\cos\omega_c t \tag{1}$$

where

$$I(t) = \sum_n (2C_{2n} - 1)rect(\frac{t}{2} - nT_c)$$
$$Q(t) = \sum_n (2C_{2n+1} - 1)rect(\frac{t}{2} - nT_c - \frac{T_c}{2}). \tag{2}$$

Here, $C_n \in \{0,1\}$ represents the nth chip, and $rect()$ function stands for the rectangle window ranging from 0 to T_c. The received signal $S_R(t)$ is the superposition of the original signal and K replicas. Based on Eq. (1), the signal $S_R(t)$ equals to

$$S_R(t) = \sum_{i=0}^{K} A_i S_{msk}(t - \tau_i) \tag{3}$$

where A_i and τ_i depict the amplitude and phase offset of the ith transmitted signal. After the coherent demodulation, the receiver decodes chips at critical decision time points

$(2n+1)T_c$ $(n=0,1,2,\ldots)$. The in-phase component of the received signal $I_R(t)$ can be decoded as

$$I_R(t)|_{t=(2n+1)T_c} = (2C_{2n}-1)\sum_{i=0}^{K} A_i \cos\omega_c\tau_i. \quad (0 \le \tau_i \le T_c) \tag{4}$$

Similarly, the orthogonal-phase component of the receiving signal $Q_R(t)$ at the decision time points $(2n+2)T_c$ can be acquired as

$$Q_R(t)|_{t=(2n+2)T_c} = (2C_{2n+1}-1)\sum_{i=0}^{K} A_i \cos\omega_c\tau_i. \quad (0 \le \tau_i \le T_c) \tag{5}$$

Eq. (4) and Eq. (5) indicate that, if delayed offsets of K replicas are less than one chip period T_c, the decoded chips are exactly the same as the transmitted chips. However, if the delayed offsets don't satisfy this constraint, delayed replicas will interfere with the original signal, influence decoding the in-phase component or the orthogonal-phase component, and bring about bit errors. The results obtained by theoretical analysis obtained from Eq. (4) and Eq. (5) match that of waveform analysis observed in Fig. 2 and Fig. 3. All those results illustrate the root cause of the precondition of constructive interference.

In order to quantitatively measure the improved reception performance due to constructive interference, we define IGF Π_R as the power gain of demodulated signals. Based on this definition, Π_R can be obtained from Eq. (4) and Eq. (5) that

$$\Pi_R = \sum_{i=0}^{K} (A_i \cos\omega_c\tau_i)^2. \tag{6}$$

According to [19] and Eq. (6), the bit error rate (BER) of a MSK modulation signal is given by

$$P_e = Q(\sqrt{2\Pi_R \frac{S}{N}}) \tag{7}$$

where $\frac{S}{N}$ represents the signal noise ratio of the received signals with Gaussian white noise and the Q function is the tail probability of the standard normal distribution [20]. Remembering that the 16 hamming mapping sequences of IEEE 802.15.4 radio can correct 8 bit errors of decoded bit streams, hence, given BER, symbol error rate (SER) could be calculated as

$$P_s = \sum_{i=9}^{32} C(32,i)(1-P_e)^{32-i}P_e^i. \tag{8}$$

For a packet with a length of l symbols, PRR could be derived from Eq. (8)

$$PRR = 1-(1-P_s)^l. \tag{9}$$

We further simulate a simplified IEEE 802.15.4 radio framework to validate theoretical waveform analysis and the results are shown in Fig. 4. An original signal and three replicas, which have relative amplitudes [1 0.5 1.5] and phase offsets $[0.25\ 0.5\ 0.75]T_c$, are superposed to a common receiver. Packets with length 4 and 64 are used to verify the performance of different system settings. To show the contribution of constructive interference, the original signal without any replica

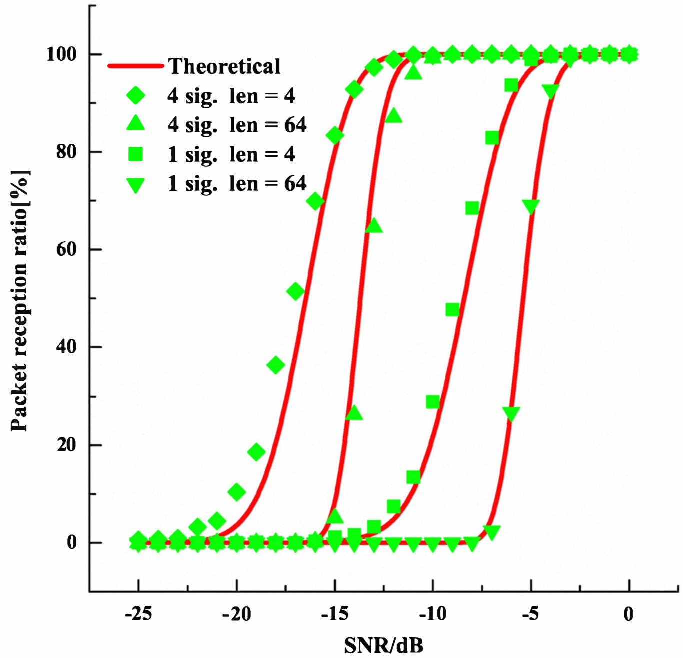

Fig. 4. PRR versus SNR for 802.15.4 radio w(o) constructive interference

is also simulated. All simulation results are averaged by 1000 times to compare with theoretical results obtained by Eq. (6) and Eq. (9). From Fig. 4, it can be observed that curves generated by theoretical analysis matches with that acquired by simulations very well. Therefore, simulations results validate the correctness of the closed-form formula Eq. (9). For both settings of transmission packet lengths 4 and 64, the measured IGF values are about 9dB, equaling to that obtained by Eq. (6). This indicates that the reception performance gain due to constructive interference is determined by relative amplitudes and phase offsets of replicas with the original signal. Furthermore, both of theoretical analysis and simulation results show that packets of longer length are much more easily corrupted by external interferences.

IV. Scalability Problem

Traditionally [7] [11], to propagate a packet across the entire network, intermediate nodes use CSMA/CA protocol to avoid potential packet collisions. However, due to carrier sense and random back-off mechanism, the protocol results in high network flooding latency, which is especially intolerable in high density and large-scale networks. Constructive interference based flooding schemes, e.g., Glossy [1] make simultaneous transmissions of packets with the same contents interfere constructively. In Glossy, intermediate nodes forward overheard packets immediately after receiving them. They trigger more nodes to receive the packets simultaneously, and the latter also start to relay the same packets concurrently. Glossy also defines a packet relay counter c and increases c by 1 before initiating a new round transmission. By taking considerable care to transmit data packets with precise timing, Glossy exploits constructive interference by quickly propagating a packet from the sink node to all the other nodes across the entire network. The time slot T_{slot} between each hop includes the durations for data reception and retransmission. The slot is determined by the packet length and thus is a network-wide constant. In this way, Glossy reaches near-optimal flooding latency.

However, it is difficult to keep precise timing for multiple concurrent transmitters in practice. We define τ_e as the time uncertainty during the time slot T_{slot} in each hop. In Glossy, τ_e is determined by the statistical uncertainty of the software delay τ_{sw}, the radio processing uncertainty τ_d, the clock uncertainty τ_{tx} due to clock frequency drifts during the packet transmission, and the propagation delay uncertainty τ_p. Therefore, we can obtain

$$\tau_e = \tau_{sw} + \tau_d + \tau_{tx} + \tau_p. \tag{10}$$

After h hops packet transmissions, the accumulated maximum time displacement Δ among concurrent transmissions to a common receiver is likely to exceed the threshold period T_c, giving rise to collisions. Meanwhile, as the number m of concurrent transmitters grows, the probability that the maximum time displacement Δ exceeds the threshold period T_c among simultaneous transmissions also rises. The above analysis shows that the increase of both h and m will bring about a reception of packet collisions, which indicates that constructive interference based flooding suffers the scalability problem. In other words, as the density or the size of a wireless network grows, the precondition $\Delta \leq T_c$ might not hold, incurring packet collisions.

For an arbitrary topology G_a, we assume that the maximal degree of all the nodes is $m \geq 2$ and the largest hop number from the sink node is $h \geq 2$. For a network topology G_w of worst case, the sink node floods packets across m independent paths, each of which includes h hops. The m independent paths join again at a common receiver R_w. For any node R_a in topology G_a, if we suppose all the other conditions are the same, we can show that the statistic reception performance of R_w in topology G_w is better than that of R_a. To analyze the reception performance of R_w, we first examine the probability mass function (pmf) of time uncertainty τ_e in one hop.

In Eq. (10), τ_p can be completely eliminated when accurate node localization is enabled. The software delay uncertainty τ_{sw} represents the additional variation due to the unsynchronized clocks of the MCU and the radio. Consequently, τ_{sw} is a discrete random variable with granularity $1/f_r$, where $f_r = 8M$Hz is the clock frequency of the radio. It should be noticed that τ_{sw} can be perfectly removed with the new generation chips e.g. cc2530, integrating MCU and radio modules in one chip with synchronized clock frequency. Caused by the offset between the asynchronous radio clocks of the transmitter and the receiver, the radio processing uncertainty τ_d is a random variable with uniform distribution in the interval $[0, 1/f_r]$. Clock uncertainty τ_{tx} during a packet transmission results from the clock frequency drifts, which are due to temperature and aging effects. In [21], the frequency drift ρ relative to the nominal frequency f_0 can be modeled as a Gaussian variable with distribution $N(0, \delta_\rho^2)$. It is reasonable to assume ρ is constant during a packet transmission time T_{slot}. Therefore, the clock uncertainty τ_{tx} due to the clock frequency drifts can be calculated as

$$\tau_{tx} = \int_0^{T_{slot}} (\frac{f_0}{f_0(1-\rho)})dt - T_{slot} \approx \rho T_{slot}. \tag{11}$$

As a result, the pmf p_e of the time uncertainty τ_e per hop can be calculated as the convolution of the pmfs of the aforementioned independent random variables.

For a path of h hops, the probability mass function (pmf) of accumulated time uncertainty τ_e^h can be obtained by

$$p_e^h = \overbrace{p_e * \ldots * p_e}^{h}. \tag{12}$$

For m independent paths, each of which consists h hops originated at the sink node, the maximum temporal displacement Δ is defined as $\Delta \triangleq \max(\tau_e^h) - \min(\tau_e^h)$. The calculation of cumulative distribution function (CDF) of Δ corresponds to the problem of finding the CDF of the range of m independent identically distributed (i.i.d.) random variables. This is a well-known order statistics problem, and the pmf has been shown in [22].

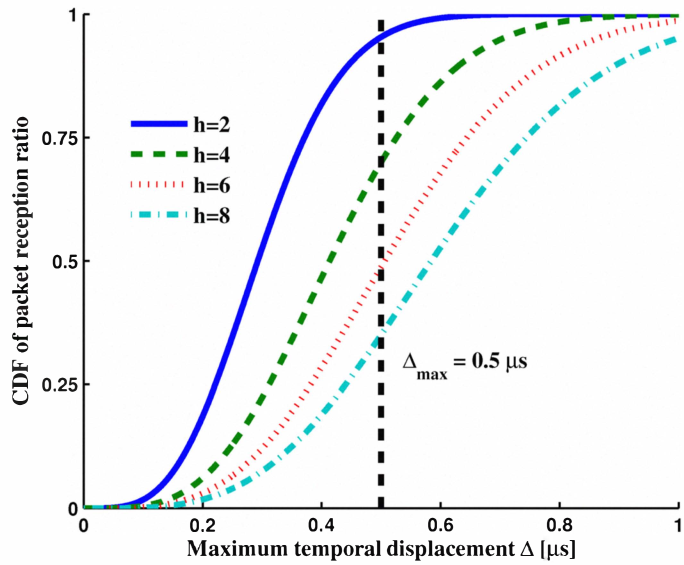

Fig. 5. CDF versus Δ of different h $(m = 5, N = 1)$.

Fig. 5 illustrates the CDF of the maximum temporal displacement Δ when the density or the size of the network of worst case varies. We choose system settings of packet length 32 and clock frequency drift variance $\delta_\rho = 5$ppm. Since flooding a same packet many times certainly increases the reliability of correct packets receptions, we require retransmission times $N = 1$ for fair performance evaluation. From Fig. 5, it can be observed that the CDF with $m = 5, h = 6$ is only 50%, which is intolerable for system design, not to mention the size or the density of the network increases. Indeed, in practice, the PRR might be better than the above worst case analysis. Increasing retransmission times N can also alleviate packet collisions problem to some extent. However, the above analysis indicates that constructive interference based flooding suffers the scalability problem, which should be addressed.

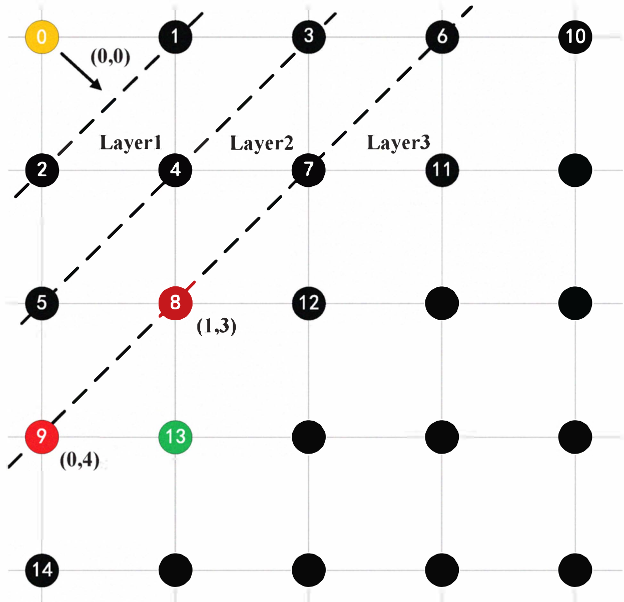

Fig. 6. Constructive interference based flooding in a 4×4 grid topology

V. Scalable Flooding with Constructive Interference

A. Constructive Interference based Flooding in Grid Networks

The aforementioned theoretical analysis provides a hint that the disclosed scalability problem has intimate relationship with the network topology. To address this problem, as a special case, network flooding in the grid topology is first analyzed. To help explain the packet propagation process, the term 'slave' stands for a receiver node. Since a slave node might have multiple parent transmitter nodes, the term 'master' represents the transmitter node dispatching the packet at the earliest time, while the term 'assistant' stands for the concurrent transmission nodes improving the packet reception. To start with, we make two general assumptions. First, the very moment when a slave receives a packet is determined by its master. Second, since a slave has at most two parent transmitter nodes in a grid topology network, the possibility for either parent transmitter becomes a master is $1/2$.

Fig. 6 illustrates constructive interference based network flooding process in a simple 4×4 grid topology. At first, the sink node N_0 broadcasts a packet to one hop slave nodes N_1 and N_2 at layer 1. After nodes N_1 and N_2 successfully receive the packet, they forward the packet immediately and simultaneously to nodes at layer 2, so on and so forth. Considering node 13, its PRR equals to the CDF of maximum temporal displacement of packet transmissions between its parent transmitter nodes N_8 and N_9. It is noteworthy that, throughout this paper, we do not consider successful packet receptions due to destructive interference. We also do not study packet collisions due to bursty links and node failures such as receiver queue overflow, packets duplicate suppression, task failure of operating systems, etc. Slave node N_8 also has two parents N_4 and N_5, each of which has $1/2$ possibility to become the master of N_8. If node N_5 becomes the slave node N_8's master, nodes N_8 and N_9 have the same master N_5, which forms the two hops independent path $N_5 \rightarrow \{N_8, N_9\} \rightarrow N_{13}$. With similar analysis, we can also obtain the three hops independent path $N_2 \rightarrow \{N_4, N_5\} \rightarrow \{N_8, N_9\} \rightarrow N_{13}$ and the four hops independent path $N_0 \rightarrow \{N_1, N_2\} \rightarrow \{N_4, N_5\} \rightarrow \{N_8, N_9\} \rightarrow N_{13}$. The three independent paths cover all the circumstances that a common ancestor node floods a packet to node N_{13} through its parent nodes N_8 and N_9. Consequently, the CDF of the maximum temporal displacement $\Delta \leq 0.5\mu s$ between N_8 and N_9 equals to the summation of the CDF in each independent case. Let $\Gamma_m^h(\Delta \leq t)$ be the CDF of the maximum time displacement Δ of a common ancestor node propagates a packet along m concurrent paths with h hops. With the same parameter settings as in section IV, the CDF of $\Delta \leq 0.5\mu s$ between node N_8 and node N_9 can be acquired as:

$$\Gamma = \frac{1}{2}\Gamma_2^1(0.5) + \frac{1}{4}\Gamma_2^2(0.5) + \frac{1}{4}\Gamma_2^3(0.5) \approx 99.23\% \qquad (13)$$

if the length of the transmission packet is 32, $\Gamma_2^1(0.5)$ can be computed with theoretical analysis in section IV. The PRR 99.2% of node N_{13} indicates constructive interference based flooding is approximately reliable.

The following analysis calculates the packet reception performance for a node far way from the sink node. Without loss of generality, an unbounded grid topology is considered and a representative pair of nodes N_W and N_Y is selected, as illustrated in Fig.7.

The L_1(Manhattan) distance, d_1^2, is defined as the summation of the lengths of the projections of the line segment between the points onto the coordinate axes in 2-dimension. For any path starting from node P to node Q along the edges in a grid network, the path is named as a L_1 path of nodes P and Q (expressed by $\chi_L(P,Q)$), if the length of the path equals to Manhattan distance $d_1^2(P,Q)$. If two L_1 paths have no intersects in between (except for the end nodes), they are called a *disjoint* L_1 path pair. The number of L_1 path between nodes P and Q is defined as $P_L(P,Q)$. The number of disjoint L_1 path pairs from node P to node Q and node R to node S is denoted as $O_L(P,R;Q,S)$. If node P and node R are the same nodes, $O_L(P;Q,S)$ is defined as the number of disjoint L_1 path pairs belonging to node P. The following lemmas can be obtained.

Lemma 1. *In Fig. 7, if $d_1^2(A,Y) = d_1^2(A,W)$, the number $O_L(A;W,Y)$ of disjoint pairs from node N_A to nodes N_W and N_Y satisfies*

$$1 \leq \frac{O_L(A;W,Y)}{O_L(B;W,Y) + O_L(C;W,Y)} \leq \frac{3}{2}. \qquad (14)$$

Proof: For any disjoint L_1 path pair $(\chi_L(B,W), \chi_L(B,Y))$ of node N_B, the L_1 path $\chi_L(B,W)$ traverses node N_D. Meanwhile, the L_1 path $\chi_L(B,Y)$ passes through node N_E. Otherwise, there will be an intersection between the path pair $(\chi_L(B,W), \chi_L(B,Y))$. Changing the head N_B of L_1 path $\chi_L(B,Y)$ to node N_C, we can obtain another disjoint L_1 path pair $(\chi_L(B,W), \chi_L(C,Y))$. Obviously, the disjoint L_1 path pair

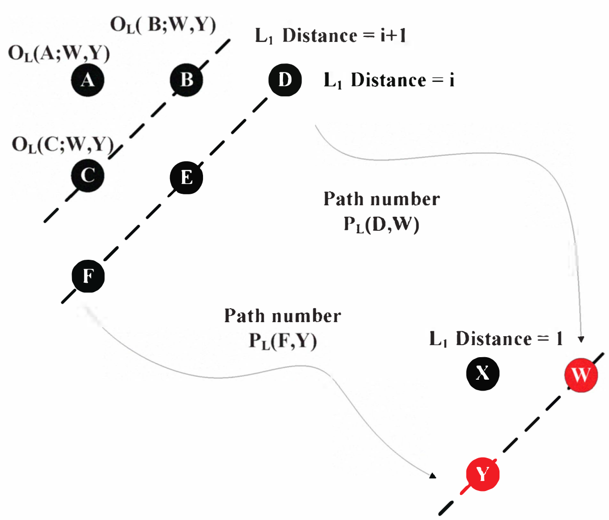

Fig. 7. Constructive interference based flooding in an unbounded grid topology: nearby parent nodes dominate the successful packet reception

$(\chi_L(B,W),\chi_L(C,Y))$ is also a disjoint L_1 path pair of node N_A, if a head N_A is added to each path of $(\chi_L(B,W),\chi_L(C,Y))$. Similarly, for any disjoint L_1 path pair $(\chi_L(C,W),\chi_L(C,Y))$ of node N_C, it is also a disjoint L_1 path pair of node N_A. For a distinct disjoint L_1 path pair $(\chi_L(D,W),\chi_L(F,Y))$, it doesn't belong to either node N_B or node N_C. However, when a head N_B is added to $\chi_L(D,W)$, and N_C to $\chi_L(F,Y)$, a new disjoint L_1 path pair $(\chi_L(B,W),\chi_L(C,Y))$ of node N_A is acquired. Indeed, we can derive the equation $O_L(A;W,Y) = O_L(B;W,Y) + O_L(C;W,Y) + O_L(D,F;W,Y)$. Accordingly, we have the inequation $O_L(A;W,Y) \geq O_L(B;W,Y) + O_L(C;W,Y)$.

For any disjoint L_1 path pair $(\chi_L(D,W),\chi_L(F,Y))$, there exists a corresponding L_1 path $\chi_L(E,Y)$ disjointing with $\chi_L(D,W)$. The corresponding disjoint L_1 path pair $(\chi_L(D,W),\chi_L(E,Y))$ belongs to node N_B. Therefore, $O_L(D,F;W,Y) \leq O_L(B;W,Y)$. In the same way, the inequation $O_L(D,F;W,Y) \leq O_L(C;W,Y)$ could be obtained. Thus, the inequation $O_L(A;W,Y) \leq 3/2(O_L(B;W,Y) + O_L(C;W,Y))$ is derived. ■

Lemma 2. *For any node P at layer $i+1$, the L_1 distance satisfies: $d_1^2(P,W) = d_1^2(P,Y) = i+1$. Let $\Phi_{i+1}(W,Y)$ represent the aggregated probability that nodes W and Y receive a packet from all possible common ancestors at layer $i+1$, then $\Phi_{i+1}(W,Y) \leq \frac{3}{4}\Phi_i(W,Y)$.*

Proof: For nodes W and Y receiving a packet from a common ancestor node N_C at layer $i+1$, which is $i+1$ L_1 distance away, the probability $\Theta_{i+1}(C;W,Y)$ equals to $O_L(C;W,Y)\big/2^{2(i+1)}$. The total probability with all common nodes at layer $i+1$ can be calculated as

$$\Phi_{i+1}(W,Y) = \sum_k \Theta_{i+1}(X_k;W,Y)$$
$$\leq \frac{1}{2^{2i+2}} \sum_k \frac{3(\Theta_i(X_k;W,Y)+\Theta_i(X_{k+1};W,Y))}{2}$$
$$\leq \frac{3}{4}\sum_k \Theta_{i+1}(X_k;W,Y) = \frac{3}{4}\Phi_i(W,Y). \quad (15)$$

■

The aggregated probability of nodes W and Y receiving a packet from all possible common transmitters in a general unbounded grid network equals to

$$\Phi_t = \sum_{i=1}^{\infty}\Phi(i;W,Y) \leq \sum_{i=1}^{8}\Phi(i;W,Y) + 3\Phi(8;W,Y)$$
$$= \frac{1}{2^2} + \frac{1}{2^4}\bullet 2 + \frac{1}{2^6}\bullet 5 + \frac{1}{2^8}\bullet 14 + \ldots \approx 0.664. \quad (16)$$

Lemma 3. *For nodes in a grid topology under constructive interference based flooding with IEEE 802.15.4 radio, if the flooding packet length is 32, the CDF of the maximum temporal displacement $\Delta \leq T_c$ between their master and assistant nodes has a lower bound 95.4%.*

Proof: For constructive interference based flooding with IEEE 802.15.4 radio, since nodes near the sink node have better PRR performance than remote nodes, we only need to analyze the PRR performance of remote nodes. For a remote node P in a grid topology, its packet reception performance can be approximated as the scenario in an unbounded grid topology. With the same parameter settings as in section IV, the CDF of the maximum temporal displacement $\Delta \leq T_c$ between the node's master and assistant can be similarly calculated as Eq. (13). Note that, the division of Φ_t shows the conditional probability of packet receptions from all possible common transmitters in the grid topology. Therefore, the CDF Γ_P is given by

$$\Gamma_P = \frac{1}{\Phi_t}\left(\frac{1}{2^2}\Gamma_2^1 + \frac{1}{2^4}2\Gamma_2^2 + \frac{1}{2^6}\Gamma_2^3 + \frac{1}{2^8}\Gamma_2^4 + \ldots\right) \approx 95.4\%. \quad (17)$$

■

Thus, lemma 3 indicates that, for constructive interference based flooding in a grid topology, no matter how many hops a node is from the sink node, it has at least 95.4% probability of successful packet reception. The implications of the conclusion are threefold. First, common parent nodes can efficiently alleviate the accumulation of time displacements, which can reduce possible packet collisions for constructive interference based flooding. Although the time displacements of packets from remote common ancestors might accumulate along the flooding paths, they are eliminated by common intermediate nodes after the packet reception and retransmission process. Second, nearby parent nodes play a key role in the successful reception of a flooding packet. In the grid topology, a successfully received packet is much more probable from nearby parent nodes. Although remote common ancestors have more disjoint path pairs to the destination, those disjoint path pairs have lower probability to be chosen. Therefore, the total contribution for packet reception performance from remote common ancestors is low. Third, for constructive interference based flooding, packet propagations paths should interleave each other if possible. Multiple independent propagation paths with long hops should be avoided to reduce time displacement accumulation.

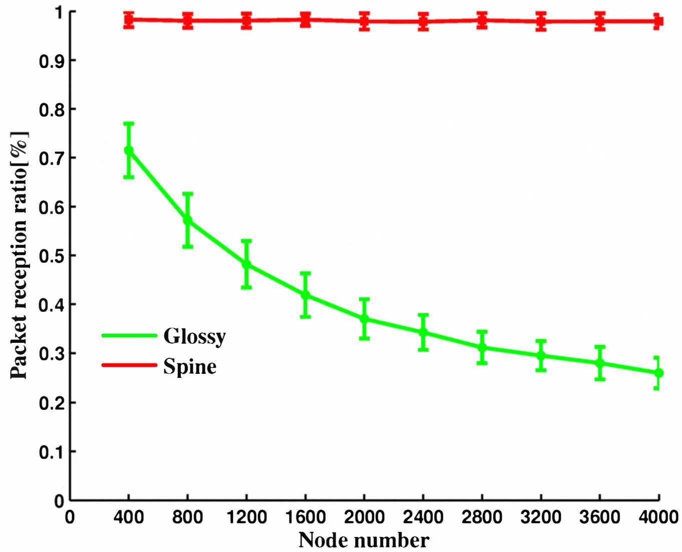

Fig. 8. PRR versus node number N_s ($l_s = 10$)

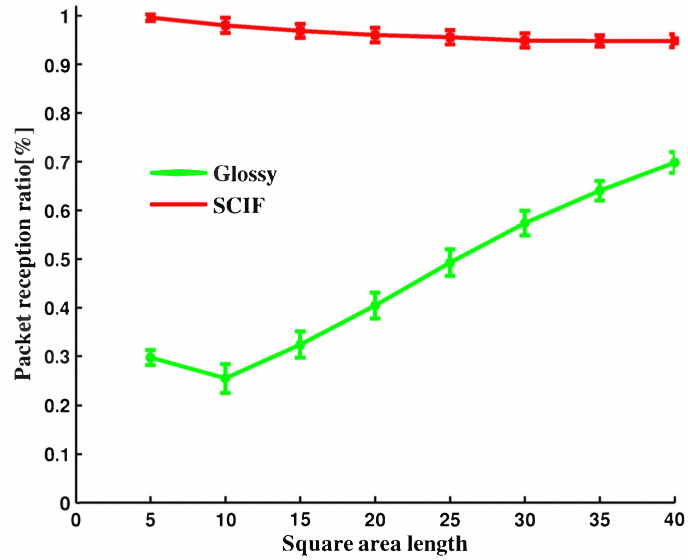

Fig. 9. PRR versus square area length l_s ($N_s = 4000$)

B. SCIF protocol

We further propose the SCIF protocol, which could be applied in a general uniformly distributed topology. For simplicity, we use the unit disk graph to model the wireless network, and assume the geographical locations of all nodes are known. We first construct a spine of a given topology through an automatic spine node selection process. We then implement network flooding on the spine in the same way as Glossy. Different from Glossy, SCIF requires that the nodes only receive the packets and keep silent, without retransmitting them again, if they do not belong to the spine. To construct the grid spine, we divide the deployment area into several grid cells, and dispatch a *CellID* ((4,4), e.g.) for each cell. According to its geometric location, each node in the network determines which cell it belongs to. One cell selects at most one node as the spine node. Therefore, nodes belonging to the same cell contend to become a spine node through an automatic process. In this process, a node firstly broadcasts a message to ask whether there exists a spine node in this cell. If not, the node waits for a certain amount of time and declares itself as the spine node for its cell [3]. Other nodes in the cell declare themselves as the dominatees of the spine node. To guarantee the spine nodes form a connecting graph, the length of a grid cell is assigned as 0.5 of the communication range. With the proposed spine construction method, a virtual grid backbone is constructed, on which constructive interference based flooding has been shown to be scalable as the density or the size of the network scales. A spine node only forwards packets when the *CellID* matches the relay counter c in the packet. In this way, SCIF ensures that overheard packets don't destroy the rhythm of the whole flooding process. After each successful packet reception, the ordinary dominatees keep silent or enter the sleep state to save energy, without forwarding the packet. The pseudocode of SCIF is illustrated in algorithm 1.

Algorithm 1: *SCIF*

```
while {!IsEmpty(& EventQueue)}{
   if {ScheduleSCIF()}{
      StartSCIF();
//construct grid spine
      SpineNodeAutoSelectByCell();
//whenever receives a flooding packet
      Do {
         StorePacket();
//keep silent if not Spine node and overheard packet
         if {IsSpineNode() && IsNotOverheard()} {
            ForwardImmediately();
         }end if
         if {SCIFComplete()}{
            StopSCIF();
            break;
         }end if
      }while {PacketReceivedEvent()}
   }end if
   DisposeOtherEvent();
   ...
}end while
Sleep();
```

VI. Simulations

To evaluate the effectiveness of the proposed SCIF protocol, we run extensive simulations in large-scale uniform distributed networks to compare the packet reception performance with Glossy, the state-of-the-art constructive interference based flooding protocol. For the fairness of the comparison, both protocols use the same theoretical model proposed in section IV. We also assume that all nodes use omnidirectional antennas and have the same transmission range. A broadcast packet can be received by nodes that are within the communication range of the transmitter. The communication range of a transmitter is assigned as 2, and nodes are uniformly distributed in a square area, with the number N_s varying from 400 to 4000 with a step of 400. The edge length of each grid cell is 1 and the length of the square area l_s varies from 5 to 40 with a step of 5. Other system parameters include: the length of packet payload 32, the variance of clock frequency drift $\delta_\rho = 5$ppm, the threshold time displacement $T_c = 0.5\mu$s (for IEEE 802.15.4 radio) and the retransmission times $N = 1$. Simulation results are averaged by 100 times, and are implemented on the Matlab7.11 platform. Fig. 8 and Fig. 9 illustrate the PRR performance as the density and the size of the network vary.

From Fig. 8 and Fig. 9, it can be observed that SCIF outperforms Glossy in terms of the PRR performance. Fig. 8 demonstrates PRR versus node number N_s with fixed deployed square area length $l_s = 10$. As N_s increases, the density of the network increases, the PRR performance of Glossy decreases significantly while that of SCIF keeps nearly constant. Particularly, when $N_s = 4000$, the PRR value of Glossy is 26% while the PRR value of Glossy is 97%. Fig. 9 shows PRR versus l_s with fixed node number $N_s = 4000$. Although the reception performance of SCIF decreases as the size of the network grows, its PRR value is higher than 96% (bounded by lemma 3, 95.4%). For Glossy, the PRR performance first drops when $l_s \leq 10$ and then increases when $l_s > 10$. We conjecture that the reason is when $l_s \leq 10$, the size of the network plays a more important role in the PRR performance than the density of the network. When $l_s > 10$, the increase of l_s adds up the sparseness of the network, and thus improves the PRR performance.

VII. Conclusion

Constructive interference based flooding is a nascent trend due to its ability to realize near-optimal network flooding latency and sub-microsecond time synchronization accuracy. With waveform analysis, we examine the root cause of constructive interference, which is previously observed in simulations and experiments only. We derive the closed-form PRR formula and define IGF to quantitatively characterize constructive interference. We disclose the scalability problem, and show constructive interference based flooding is scalable (PRR lower bound 95.4%) in the grid topology with theoretical analysis. We further propose the SCIF protocol, which outperforms Glossy in terms of the PRR performance when the density or the size of the network grows. Future work including the performance measurements of SCIF in the real world large-scale wireless sensor networks (e.g., the CitySee [6] project in Wuxi city, China, 4000+ nodes), and the exploitation of constructive interference in remote reprogramming, is under active research.

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